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Subatomic Physics
Final exam
Date: Tuesday, Feb. 1, 2005
This exam has a total of 100 points

Problem #1. *(5 points)*

Certain radioactive nuclei emit α particles (made out of 4 nucleons with a mass of roughly 3750 MeV/c^2). If the kinetic energy of these α particles is 4 MeV, what is their velocity if you assume them to be non-relativistic? (2 points)

How large an error do you make in neglecting special relativity in the calculation of this velocity? (3 points)

Problem #2. *(4 points)*

Calculate the approximate density of nuclear matter in g/cm^3. What would be the mass of neutron star that had the diameter of an orange? Note that 1 atomic mass unit is 1.66×10^{-27} kg.

Problem #3. *(4 points)*

If the stable isotope of sodium is ^{23}Na, what kind of radioactivity would you expect from (a) ^{22}Na and (b) ^{24}Na?

Problem #4. *(10 points)*

a) Given that the form factor of a nucleus can be expressed as the Fourier transform of the charge-density distribution

$$F(q) = \int d^3R \; \rho(\vec{R}) \; e^{-i\vec{q}\cdot\vec{R}} \quad .$$

and assuming a spherical shape for the charge-density distribution, perform an expansion around small momentum transfers and explain the first three terms that you obtain in terms of physical quantities (moments). *(6 points)*

b) Calculate the relation between the root-mean-square radius R_{rms} and the radius R of a homogeneously-charged sphere with charge Ze. *(4 points)*

Problem #5. *(15 points)*

a) Write down the Bethe-Weizsäcker empirical mass formula (you don't need to know the constants. Here, they are given for your information: $a_V = 15.85$ MeV, $a_S = 18.34$ MeV, $a_a = 23.21$ MeV, $a_c = 0.71$ MeV and $a_p = 12$ MeV). *(4 points)*

b) Give a physical argument as to why the form is parabolic in Z for large Z? Elaborate by making an estimation leading to this term. Don't worry about the coefficient. *(3 points)*

c) Calculate the relation between the mass number A and the charge number Z for the most stable nuclei assuming large A (line of stability). *(4 points)*

d) For a constant *odd* A, draw a picture of the binding energy as a function of Z and show how the stable isotope is reached and through which decay paths (specify the decay type as well). *(4 points)*

Problem #6. *(15 points)*
The single-particle level scheme with a Woods-Saxon potential is given in the appendix.

a) Calculate the spin and parity of the ground state and the first three excited states of ${}^{41}_{20}$Ca, ${}^{35}_{17}$Cl, and ${}^{23}_{11}$Na. *(6 points)*

b) Using the Schmidt formula for the magnetic moment $\mu_J = J g_J \mu_N$, where $g_J = g_l \pm \frac{g_s - g_l}{2l+1}$, with $g_l = 1(0)$, $g_s = 5.586(-3.826)$ for protons (neutrons), predict the value of the magnetic dipole moment of ${}^{41}_{20}$Ca. *(4 points)*

c) Determine the magnetic dipole moment μ/μ_N of the deuteron for a pure ${}^3S_1$ state. Note that deuteron is made out of one proton and one neutron which couple to $J = 1$ with spins aligned when particles are both in s orbital to form the pure ${}^3S_1$. The calculated value is about 2.5% larger than the observed value. What could we learn from this? *(5 points)*

Problem #7. *(7 points)*

a) What is the energy dependence of the beta decay spectrum in a Kurie or Fermi plot for massless neutrinos. The Kurie plot is $(\frac{d\omega}{p^2 dp})^{1/2}$ as a function of the electron energy *(4 points)*

b) What happens in this plot when the neutrino is massive? Show this on a plot (Do NOT derive the formulas). *(3 points)*

Problem #8. *(12 points)*

a) Show that a measurement of any pseudo-scalar observables must be an evidence of parity violation (initial and final states have the same parity). *(6 points)*

b) If parity is conserved, prove that the expectation value of any operator that changes sign under parity operation is zero unless the initial and final states have opposite parities. *(6 points)*

Problem #9. *(10 points)*

a) Write down the Gell-Mann Nishijima relationship relating the charge to isospin, Baryon number and strangeness. *(4 points)*

b) The Σ baryon exists in 3 charge states. Show that the strangeness quantum number of this particle is $S = -1$. *(3 points)*

c) The Ξ baryon exists in 2 charge states (-1 and 0). Show that the strangeness quantum number of this particle is $S = -2$. *(3 points)*

Problem #10. *(6 points)*

The Ξ^- has $J^P = \frac{1}{2}^+$. It decays through weak interaction into a Λ^0 and a π^- meson. If $J_\Lambda^P = \frac{1}{2}^+$ and $J_\pi^P = 0^-$, what are the allowed relative orbital angular momenta for the $\Lambda - \pi^-$ system? Show how you obtain your results.

Problem #11. *(12 points)*

Which of the following reactions are allowed and which are forbidden? If allowed, explain which interaction and if not, why not? Write down all the conserved or violated conservation laws relevant for the reaction. *(2 point for each reaction)*

1. $\Lambda \rightarrow p + e^- + \bar{\nu}_e$
2. $\Lambda \rightarrow p + e^-$
3. $\bar{p} + n \rightarrow \pi^- + \pi^0$
4. $p + p \rightarrow K^+ + K^+ + n + n$
5. $n \rightarrow p + \pi^-$
6. $\Sigma^+(1385) \rightarrow \Lambda^0 + \pi^+$

Here are the quantum numbers of the particles involved in these reactions:

- $K^\pm$: $I(J^P) = \frac{1}{2}(0^-)$ $S = -1$ for K^-
- $\pi^\pm$: $I^G(J^P) = 1^-(0^-)$
- π^0: $I^G(J^{PC}) = 1^-(0^{-+})$
- p: $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$
- n: $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$
- Λ^0: $I(J^P) = 0(\frac{1}{2}^+)$ $S = -1$
- $\Sigma^{\pm,0}$: $I(J^P) = 1(\frac{1}{2}^+)$ $S = -1$
- Ξ^0: $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ $S = -2$

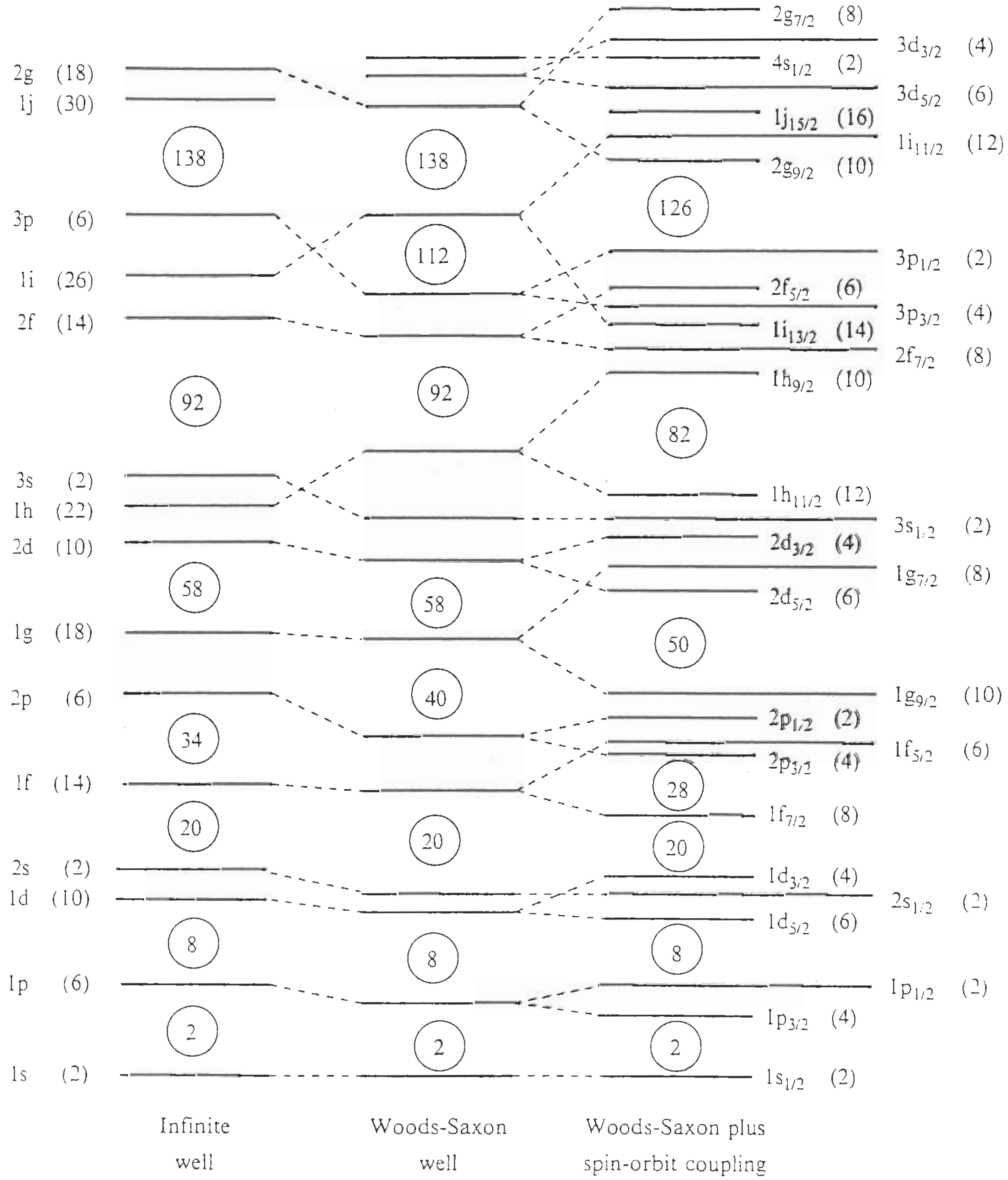

Figure 2.9 Sequences of bound single-particle states calculated for different forms of the nuclear shell-model potential. The number of protons (and neutrons) allowed in each state is indicated in parentheses and the numbers enclosed in circles indicate magic numbers corresponding to closed shells.